An approch for Stable Neo-Hookean Flesh Simulation

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ABSTRACT

This paper introduces a new method for fleshy simulation, by obtaining the closed-form expressions of eigenvalues and eigenvectors of all components of the system to directly project the Hessian to semi-positive-definiteness. This is a new version of the Neo-Hookean elastic model, which maintains the fleshy appearance of the Neo-Hookean model. It helps to deeply understand the numerical behavior of materials, exhibits superior volume preservation, and is robust to extreme kinematic rotation and inversion. We made an improvement based on assignment 3 (finite element 3D model), which uses the finite element method under the tetrahedral structure. We mainly optimize strain energy formulas and use stable Neo-Hookean elasticity to achieve the optimization of hyperelastic energy density. These findings also provide information for the design of more complex hyperelastic models, and we provide an extensive comparison with existing material models.

Author Keywords

Physically-based Simulation, Elasticity

CCS Concepts

• **Computing methodologies → Physical simulation;**

ACM Reference Format:

Breannan Smith, Fernando de Goes, and Theodore Kim. 2018. Stable Neo- Hookean Flesh Simulation. ACM Trans. Graph. 37, 2, Article 12 (March 2018), 15 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

INTRODUCTION

For visual characters, we should carefully choose elastic energy, since it determines the visual quality of a simulation for model deformations. Volume preservation is the defining quality for biological tissues such as muscle and fat, selecting in their high Poisson ratios $\nu \in [0.45, 0.5)$ [Greaves et al. 2011]. Most materials have Poisson's ratio values ranging between 0.0 and 0.5. However, for soft materials like rubber, where the bulk modulus is much higher than the shear modulus, Poisson's ratio is near 0.5.

However, it is well known that this approximately incompressible region is difficult to simulate reliably and accurately, and many existing production techniques still can't figure it out. For instance, the popular co-rotational model for elasticity [Chao et al. 2010; McAdams et al. 2011; Müller et al. 2002; Nesme et al. 2009] ostensibly supports materials within this range, but it fails to preserve volume, instead, it collapses the trapezius muscle and forms false folds around the scapula. The co-rotational model for elasticity linearizes the volume term through a compromise between its volumetric properties and its visual results. Moreover, force filtering approaches [Irving et al. 2004; Teran et al. 2005] introduce additional user parameters although they inherit the visual quality of the Neo-Hookean model.

Therefore, we propose a novel Neo-Hookean energy that retains the rich volume characteristics of biological materials and does not need to introduce additional filter parameters. We applied our method on assignment 3 to optimize the finite element method since FEM is difficult to analyze nonlinear systems.

Our model exhibits better volume preservation when compared to the FEM model in assignment 3, and performs well in a wide range of Poisson's ratios $\nu \in [0, 0.5)$. Additionally, it is robust to extreme kinematic rotation and inversion. At present, our model is better than all existing models and has more advantages, so it is very competitive.

We obtain these properties by analyzing the Hessian function of energy and obtaining the closed-form expressions of eigenvalues and eigenvectors of all components of the system. Then the eigendecomposition can be written into a set of compact expressions and project them back to semi-definiteness and isolate indefiniteness. Therefore the energy can be safely applied to a Newton-type implicit integration scheme, which is a linear solver based on conjugate gradient.

Although we are driven by the Neo-Hookean model, which is relatively unpopular with other models, it is general enough and supports any model calculated according to the first and third invariants of the deformation gradient.

As a preliminary example, we showed how to extend it to assignment 3(finite element method). Then we compare the existing hyperelastic energies in detail.

RELATED WORK

The research on flesh simulation has spanned decades, but most models can not realize preserve volume or are too complex that require many parameters. Therefore it has no direct relationship with our method. A major disadvantage of nonlinear hyperelastic energies is that they usually contain singularities. Early work on deformable simulation [Hirota et al. 2001] detected these "illegal" states and carefully avoided them using backtracking line search, but these searches are

ISBN 0730-0301/2018/3-ART12...$15.00
DOI: https://doi.org/10.1145/nnnnnnn.nnnnnnn

Symbol Definition

F=RS	*polar decomposition*
J = det(F)	*Relative volume change*
C = F^TF	*Right Cauchy-Green tensor*
I_C *= tr (C)*	*First Right Cauchy-Green invariant*

Table 1. Quantities derived from the deformation gradient F

difficult to perform robustly. In Section 4, we compare our work with the finite element method in assignment 3.

METHOD

For the project, we use a tetrahedron as the basic space volume. Therefore we apply finite element method upon it. We extract the 4D vector which contains the values of the shape functions for each vertex at the world space point x (assumed to be inside the element).

Also, we use hyperelastic energy density Ψ To represent the elastic behavior of deformable bodies and use the first Piola-Kirchhoff (PK1) stress tensor to derive force. In assignment 3, we use $\Psi_B = \frac{\mu}{2}(J^{-\frac{2}{3}}I_C - 3) + \frac{\lambda}{2}(J-1)^2$ [Bower 2009], and we improve the flesh simulation by using $\Psi_D = \frac{\mu}{2}(I_C - 3) + \frac{\lambda}{2}(J-1)^2$, which can achieve inversion stability, reflection stability, rest stability, and meta-stability under degeneracy. Please refer to table1 for the quantities derived from the deformation gradient F.

Moreover, we use the skinning to translate the motion from low resolution armadillo tetrahedron mesh to the high resolution rendering mesh in the file $build\_skinning\_matrix.cpp$.

Existing Neo-Hookean Energies

There are few version of Neo-Hookean elasticity:

(1) $\Psi_{Neo} = \frac{\mu}{2}(I_C - 3) - \mu logJ + \frac{\lambda}{2}(logJ)^2$[Bonet and Wood 2008]

(2)$\Psi_A = \frac{\mu}{2}(I_C - 3) - \mu logJ + \frac{\lambda}{2}(J-1)^2$ [Ogden 1984]

(3)$\Psi_B = \frac{\mu}{2}(J^{-\frac{2}{3}}I_C - 3) + \frac{\lambda}{2}(J-1)^2$ [Bower 2009]

(4) $\Psi_C = \frac{\mu}{2}(J^{-\frac{2}{3}}I_C - 3) + \frac{\lambda}{2}(J-1)$ [wang and And 2016]

Equation (1) is the most common version, and assignment 3 uses equation (2). We examine these energies according to the Valais Landel hypothesis [Xu et al. 2015], which assumes that many hyperelastic energies can be divided into length (1D), area (2D), and volume (3D) components. The above model includes length and volume terms but does not contain area terms.

Modified Neo-Hookean Energies

1D Length Term: Mooney [1940] originally proposed the energy which was later dubbed the "Neo-Hookean" energy by Rivlin [1948]:

(5)$\Psi_M = \frac{\mu}{2}(I_C - 3)$

Equation (5) has the hard constraint that J = 1, therefore the energy is minimized at the volume holding configuration closest to the origin of the stretched space. Also Ψ_M performs well under inversion. The energy relative to the zero volume configuration is always clearly defined, independent of the current state of the element. However,Ψ_B and Ψ_C introduces the numerical problem of unbounded growth under compression, it has the limitation that it will become undefined when J = 0. So Ψ_M is our choice for a modified energy. And that's how we improve finite element method in assignment 3.

3D Volume Term:

(6) $\Psi_{Neo,volume} = -\mu logJ + \frac{\lambda}{2}(logJ)^2$

It becomes infeasible for $J < 0$ except growing unbounded as $J \to 0$, however, any algorithm with a logoarithm will have the same problem. So we will not consider $\Psi_{A,volume} = -\mu logJ + \frac{\lambda}{2}(J-1)^2$, and instead $\Psi_{B,volume} = \frac{\lambda}{2}(J-1)^2$ is bounded, well-defined, and invertible, and these terms have been used [Martin et al. 2011; Teschner et al.2004] to avoid the need for any inversion processing.

Therefore, equation 7 is the optimize equation so far, but it has some disadvantages we should mention about.

(7)$\Psi_D = \frac{\mu}{2}(I_C - 3) + \frac{\lambda}{2}(J-1)^2$

Key Drawback of Ψ_D

$\Psi_D = \frac{\mu}{2}(I_C - 3) + \frac{\lambda}{2}(J-1)^2$ lacks rest stability although it achieves inversion stability. Ψ_D satisfies hyperelastic energy vanish at identity, however, it doesn't ensure that $PK1$ resolves to 0, which is a true indicator that the energy has reached its extreme value. Therefore it will lead to the problem that when the body is at rest, a force occurs and the expected rest state is overwritten with different parameter-related states. If we reduce the visual appearance of artifacts by greatly increasing λ(e.g. [Blemker et al. 2005] suggests 1000μ), it excludes artifact-free simulation of materials with low Poisson's ratio, and it will increase the uncertainty of the Hessian curve.

The problem is that the effective rest state coincides with a smaller J, causing an element to deflate slightly. Therefore, we can consider whether each element can be inflated to make it stable at J = 1 during deflate.

RESULT

We compared the two methods of the strain energy density. To be exact, one is $\Psi_B = \frac{\mu}{2}(J^{-\frac{2}{3}}I_C - 3) + \frac{\lambda}{2}(J-1)^2$ for assignment 3 and the modified version is $\Psi_D = \frac{\mu}{2}(I_C - 3) + \frac{\lambda}{2}(J-1)^2$. In our experiment, we can see the obvious difference. Assignment 3 can't preserve the volume and will explode because of serious deformation. In these experiments, we processed the data to illustrate the stability of the new algorithm.

Flesh Simulation

From figure 1 and figure 2, we can notice that the finite element method in assignment 3 can't maintain the fleshy appearance of the Neo-Hookean model, and the finger is deformed and cannot come back. If you pull a little harder, the visual character will explode. Instead, the new Stable Neo-Hookean model exhibits superior volume preservation and is robust to extreme kinematic rotation and inversion.

SUMMARY

In this research, we have introduced the new version of the new-Hooke elastic model from modifying the existing model.

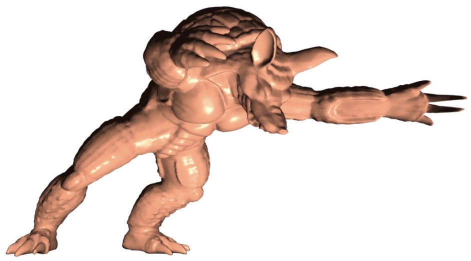

Figure 1. Operation of Assignment 3: Unstable

Figure 2. Operation of Assignment 3: exploded

Figure 3. Stable Neo-Hookean Flesh Simulation:front angel drag

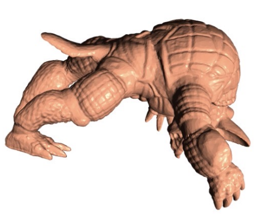

Figure 4. Stable Neo-Hookean Flesh Simulation:side angel drag

We combine the advantages of the existing model to implement it, like the Strain Energy density function used by assignment 3. We obtain the closed-form expressions of eigenvalues and eigenvectors of all components of the system to directly project the Hessian to semi-positive-definiteness. Moreover, our model has excellent volume-preserving properties and we will improve it further by solving the problem in section 4.3. Please see the authors' websites for additional supplementary material.

ACKNOWLEDGEMENT

Wenqing Cao acknowledges helpful discussions with Haonan Gao.

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